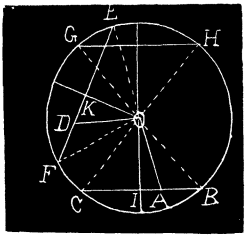
pose the second and fourth to be taken somewhere upon the lines BC and EF, respectively.

Put $OC=a$, $OA=x$, $OD=y$, $\angle OAC=\theta$, $\angle IOC=\phi$, $\angle ODK=\psi$, and $\angle KOF=\omega$.



Then $a\cos\phi=x\sin\theta$, and $a\cos\omega=y\sin\psi$.

If either E or F is upon the arc HG, the quadrilateral formed by joining the extremities of the chords will contain the center of the circle.

Since the arc HG=the arc CB, the probability of this is $(\phi+\omega)/\pi$.

If $x>0$ and $<a$ the chance that the first point is taken between x and $x+dx$ is $2\pi xdx/a^2\pi=2xdx/a^2$.

If $y>0$ and $<a$ the chance that the third point is taken between the distances y and $y+dy$ is $2ydy/a^2$.

If $\theta>0$ and $<\frac{1}{2}\pi$ the chance that the second point is taken between the line BC and a second line making at A the angle $d\theta$ is $\frac{1}{2}(AC^2+AB^2)d\theta/\frac{1}{2}a^2\pi$ $=2[a^2+x^2(1-2\sin^2\theta)]d\theta/a^2\pi=2(1-2\cos^2\phi+\cos^2\phi\operatorname{cosec}^2\theta)d\theta/\pi$.

If $\psi>0$ and $<\frac{1}{2}\pi$ the chance that the fourth point is taken between the line EF and a second line making at D the angle $d\psi$ is $2(1-2\cos^2\omega+\cos^2\omega\operatorname{cosec}^2\psi)d\psi/\pi$.

If we suppose θ constant while x and ϕ vary, $xdx=-a^2\sin\phi\cos\phi\operatorname{cosec}^2\theta d\phi$. When $x=0$, $\phi=\frac{1}{2}\pi$, and when $x=a$, $\phi=\frac{1}{2}\pi-\theta$. Hence the limits of integration for ϕ are $\frac{1}{2}\pi-\theta$ and $\frac{1}{2}\pi$. If we integrate first with respect to θ, the limits for θ are $\frac{1}{2}\pi-\phi$ and $\frac{1}{2}\pi$, while for ϕ they are 0 and $\frac{1}{2}\pi$.

In like manner we may substitute $-a^2\sin\omega\cos\omega\operatorname{cosec}^2\psi d\omega$ for ydy and integrate between limits $\frac{1}{2}\pi-\omega$ and $\frac{1}{2}\pi$ for ψ, and between 0 and $\frac{1}{2}\pi$ for ω.

Hence the required probability is

$$P=\frac{16}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\phi}^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\omega}^{\frac{1}{2}\pi}(\phi+\omega)(1-2\cos^2\phi+\cos^2\phi\operatorname{cosec}^2\theta)\times$$

$$(1-2\cos^2\omega+\cos^2\omega\operatorname{cosec}^2\psi)\sin\phi\cos\phi\operatorname{cosec}^2\theta\sin\omega\cos\omega\operatorname{cosec}^2\psi\, d\phi\, d\omega\, d\theta\, d\psi$$

$$=\frac{256}{9\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}(\phi+\omega)\sin^4\phi\sin^4\omega\, d\phi\, d\omega$$

$$=\frac{4}{9\pi^3}\int_0^{\frac{1}{2}\pi}(12\pi\phi+3\pi^2+16)\sin^4\phi\, d\phi=\tfrac{1}{2}+(8/3\pi^2)=.77019.$$

MISCELLANEOUS.

55. Proposed by J. M. COLAW, A. M., Monterey, Va.

Multiply 6 by 4. Is the problem legitimate when both symbols represent pure number?

[NOTE. "A measured or numbered quantity may be divided into a number of parts, or taken a number of times; but no number can be multiplied or divided into parts."—*McLellan and Dewey's Psychology of Number*. "The astounding thesis is maintained that number is not a magnitude, does not possess quantity at all, and that 'no number can be multiplied or divided into parts'."—*Lefevre's Number and Its Algebra*.]

I. Comment by DAVID EUGENE SMITH, Ph. D., Professor of Mathematics, Michigan State Normal School, Ypsilanti, Mich.

This question seems to me to have no interest to mathematicians. It simply means that somebody has set up a narrow definition of multiplication and has then said, "*Your* work is not multiplication because it does not fit *my* definition." I expressed my humble opinion in the MONTHLY some time ago when the antiquated definition of division was brought up to prove that it was impossible to divide $10 by 2. Such narrow limitations seem to me utterly nonsensical.

In a similar sense we cannot multiply by -1, and we cannot have "imaginary numbers," and 1 is not a number, etc., etc. Mathematical progress has always been made the more difficult because somebody has insisted on hanging on to some antiquated definition.

What do these people who say that we cannot multiply 2 by 3 say to some such simple formula as $e^{\pi i}=-1$? I suppose they say that e, π, i, and -1 have no existence.

II. Comment by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

There may be a modern idea that since neither 6 nor 4 is a numbered quantity, the operation is impossible. If it were possible to get the evidence of all the mathematicians, I am sure not one could be found who did not learn his multiplication table, in fact, get his aptness in numbers by the same process as given in the problem. There may be some who claim otherwise but I would even doubt their claim.

$6\times4=24$ is good arithmetic. It seems a pity that vandals should make incursions upon the sacred shrines of Newton, La Place, Pierce, and other noted men of numbers, and so desecrate their immortal works, as to try and mistify their teachings. The God of Mathematics will not permit it.

III. Remarks by J. K. ELLWOOD, A. M., Principal of the Colfax School, Pittsburg, Pa.

Do $4\times6=24$? Can 6 be multiplied by 4? Six what? If 6 units of quantity, yes; but if not a magnitude,—well, what then is it? "Six" apart from the universe of space, time, and matter, suggests to the mind—what? The "how many"? The *ratio* of the "how many" to the unit? Six in the abstract —a pure number—can not, in an arithmetical sense, be multiplied by any abstraction. In an algebraic sense, $4\times6=24$, just as $x\times x=x^2$. That is, we operate with *symbols*, neglecting the realities represented. If two abstract numbers can be multiplied one by the other, why not two concrete numbers, as feet$\times$feet =square feet?

42. Proposed by E. B. ESCOTT, Cambridge, Mass.

To find triangles whose sides and median lines are commensurable.

II. Solutions communicated to "L' Intermediaire des Mathematiciens" (January, 1898) by the PROPOSER. Selected and translated by J. M. COLAW, A. M., Monterey, Va.

First Solution. By Chas. Gill (New York, 1848).

$x=t[1-(\cos A+\sin A)(\cos B+\sin B)]$, $y=t[\cos B-\sin B+(\cos A-\sin A)(\cos B+\sin B)]$, $z=t[\cos A-\sin A+(\cos B-\sin B)(\cos A+\sin A)$, whence there exists one of the four relations following: